Viscous transonic outflow in equatorial discs of Be stars

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Abstract. We discuss the characteristics of the outflow in discs of Be stars, based on the viscous decretion disc scenario. Solving the wind equation, we find that a transonic solution exists for any α. The sonic point is located at $r > 100R$ for plausible values of parameters, where R is the stellar radius. The sonic radius is smaller for higher temperature and/or larger radiative force. We also find that the topology of the sonic point is nodal for $\alpha \gtrsim 0.95$, while it is saddle type for $\alpha \lesssim 0.9$. In the subsonic region, the outflow velocity increases as r and the surface density decreases as r^{-2}. The angular velocity of the disc decreases as $r^{-1/2}$ in the inner subsonic region, while it decreases as r^{-1} in the outer subsonic region.
keywords: hydrodynamics – stars: circumstellar matter – stars: emission line, Be – stars: mass loss

1 Introduction

Be stars are non-supergiant early-type stars with Balmer emission lines, whose spectral types range from late O- to early A-type. Extensive studies have revealed that a Be star has a two-component extended atmosphere, a polar region and a cool disc-like region. The polar region consists of a low-density, fast outflow emitting UV radiation. The mass loss rate of this region inferred from the UV lines is about $10^{-11} - 10^{-9}\, M_\odot\ \mathrm{yr}^{-1}$ (Snow 1981).

In contrast to the polar region, the disc region consists of a high-density plasma whose outflow velocity is small. The optical emission lines and the IR excess arise from this region. Unfortunately, the nature of this region is not well understood, depsite that large observational efforts have been devoted to the study of Be stars.

Lee et al. (1991) constructed a disc model, in which matter rotates at Keplerian speed but gradualy drifts outward because of the effect of viscosity. Assuming Keplerian rotation and neglecting the advective term in the equation of momentum coservation, they obtained steady structure of viscous decretion discs around Be stars. The purpose of this paper is to study the characteristics of the outflow in viscous decretion discs around Be stars.

2 Basic equations for viscous decretion discs

We assume that the circumstellar disc of a Be star is steady, geometrically thin, and symmetric about the rotational axis and the equatorial plane. Moreover, we assume for simplicity that the disc is isothermal, and adopt the Shakura-Sunyaev viscosity prescription. Since the deviation from a point mass potential due to the rotational distortion of the star is small in general, we neglect the quadrupole contribution to the gravitational potential. We assume that the radiative acceleration arises from the absorption of stellar continuum by an ensemble of optically-thin weak lines, and adopt the parametric form for the radiative force proposed by Chen & Marlborough (1994).

From the vertically integrated equations describing the mass, momentum, and angular momentum conservation in a disc around a star of mass M and radius R, together with the equation of state and the Shakura-Sunyaev viscosity prescription, we obtain the following equations which describe the stationary viscous flow.

$$\left(V_r - \frac{c_{\rm s}^2}{V_r}\right)\frac{{\rm d}V_r}{{\rm d}r} = -\frac{GM}{r^2}\left[1-\eta\left(\frac{r}{R}\right)^{\epsilon}\right] + \frac{\ell^2}{r^3} + \frac{5}{2}\frac{c_{\rm s}^2}{r}, \tag{1}$$

$$\ell = \ell_{\rm s} + \alpha c_{\rm s}^2\left(\frac{r_{\rm s}}{c_{\rm s}} - \frac{r}{V_r}\right), \tag{2}$$

where $c_{\rm s}$ is the isothermal sound speed, α is a viscosity parameter, η and ϵ are parameters which characterize the radiative force, and V_r and ℓ are the vertically-averaged radial velocity and specific angular momentum, respectively. The subscript 's' denotes the quantities at the sonic point.

It is important to note that, in the current problem, the angular momentum distribution of the flow is not given a priori but obtained as the solution of Eqs. (1) and (2); $r_{\rm s}$ and $\ell_{\rm s}$ are the eigenvalues of these equations. Note also that Eq. (2) indicates that a tiny deviation from the Keplerian rotation can cause a drastic change in the radial velocity distribution.

3 Viscous transonic solutions

In the following examples, the central star is a B0 main-sequence star with $M = 17.8 M_\odot$, $R = 7.41 R_\odot$, and $T_{\rm eff} = 2.80 \cdot 10^4\,{\rm K}$ (Allen 1973). Since, the disc is assumed to be isothermal, the structure of the viscous transonic outflow is independent of the mass decretion rate [see Sec. 3 of Abramowicz & Kato (1989)].

3.1 Structure of the viscous transonic outflow

Figure 1 shows a typical structure of the viscous transonic decretion disc. From Fig. 1, we find that the sonic point is located far from the star and the outflow

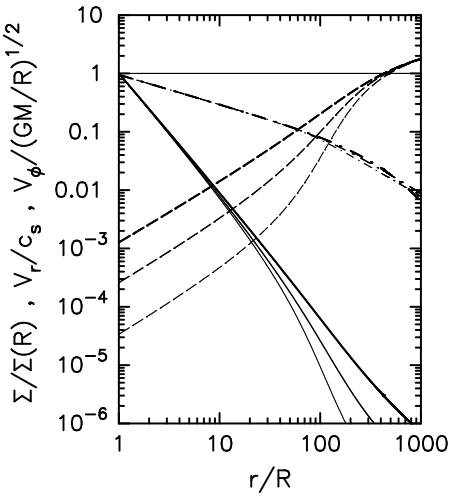

Figure 1: Structure of the viscous transonic decretion disc for $T_{\rm d} = \frac{2}{3} T_{\rm eff}$ and $(\eta, \epsilon) = (0.1, 0.1)$, where $T_{\rm d}$ is the disc temperature. Solid, dashd, and dash-dotted lines denote $\Sigma/\Sigma(R)$, $V_r/c_{\rm s}$, and $V_\phi/(GM/R)^{1/2}$, respectively. Thick lines, lines with intermediate thickness, and thin lines are for $\alpha = 1$, $\alpha = 0.1$, and $\alpha = 0.01$, respectively

is highly subsonic for $r \ll 10^2\, R$. This is because it is basically the pressure force which accelerates the flow up to a supersonic speed. Consequently, the sonic radius is smaller for higher disc temperature and/or larger radiative force.

We also find that, in the subsonic part of the disc, V_r increases as r and Σ decreases as r^{-2}. In the inner subsonic region, V_ϕ decreases as $r^{-1/2}$, while in the outer subsonic region it decreases as r^{-1}. It should be noted that these radial dependences hold in general in viscous decretion discs.

Our results, together with the observed range of the base density for Be star discs (Dougherty et al. 1994), suggest that the mass loss rate in the equatorial region is at most comparable with that in the polar region.

3.2 Topology of the sonic point

It is well known that the stability of the transonic accretion with Shakura-Sunyaev viscosity prescriptions is related to the topology of the sonic point (e.g., Abramowicz & Kato 1989, and references therein). For α greater than a critical value, the sonic point is nodal and the transonic accretion is unstable. On the other hand, for α smaller than the critical value, the sonic point is saddle type and the stable transonic accretion exists. The instability arises as a result of work done by viscous force [see Kato et al. (1988) for detailed analysis].

In Fig. 2, we present an example showing the location of the sonic points in the $(r_{\rm s}, \lambda_{\rm s})$-plane. As shown in Fig. 2, in our model, the topology of the sonic

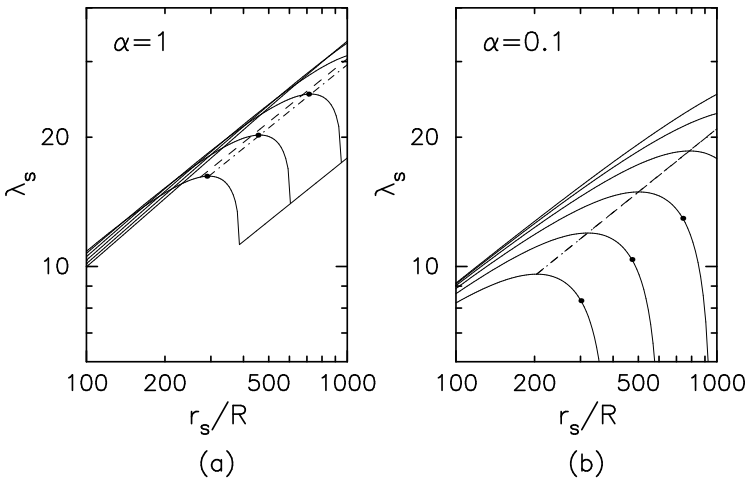

Figure 2: Diagram showing the location of the sonic points in the $(r_{\rm s}, \lambda_{\rm s})$-plane for (a) $\alpha = 1$ and (b) $\alpha = 0.1$, where $\lambda_{\rm s} = \ell_{\rm s} + \alpha c_{\rm s} r_{\rm s}$ is normalized by $(GMR)^{1/2}$. Each curve denotes the relation between $r_{\rm s}$ and $\lambda_{\rm s}$ for a constant sound speed. From the bottommost curve upwards $T_{\rm d}/T_{\rm eff} = 1.0$, 0.82, 0.64, 0.46, 0.28, and 0.10. The filled circle on each curve indicates the position of the sonic point. The dashed line separates the spiral-type sonic point region (upper side of the line) from the nodal-type sonic point region (lower side of the line), and the dash-dotted line separates the nodal-type region (upper side) from the saddle-type region (lower side). In Fig. 2(b) the nodal-type region is so narrow that it is indistinguishable on the figure. The thin line in Fig. 2(a) indicates the bounrary in the parameter space below which the circular orbit at the sonic radius is unstable

point is nodal for $\alpha \gtrsim 0.95$, while it is saddle type for $\alpha \lesssim 0.9$. This means that the stationary transonic decretion in discs of Be stars exists if $\alpha \lesssim 0.9$, whereas the transonic decretion cannot be stationary if $\alpha \gtrsim 0.95$.

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