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Theory vs. Observation of Circumstellar Disks and Their Formation

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Abstract. Several classes of active OB stars have circumstellar disks. Among them, Be stars have many unique characteristics. In this review, I first discuss key observational features of circumstellar disks around Be stars, which provide constraints on disk properties, such as the geometry, the density and temperature distributions, the velocity field, and physical processes on the disk formation and loss. Then, I discuss the disk evolution on the basis of the viscous decretion disk model, which is at present the most satisfactory model. I also show how the Be disk evolves when the mass ejection from the central star is turned off for a while and then resumed.

1. Introduction

In the original plan, John M. Porter was the speaker of this review talk. However, quite shockingly to all of us, he suddenly passed away shortly before this conference. If he had presented this talk as originally planned, he would have done it in his usually-cheerful and at the same time uniquely-insightful way, describing the key observational features of Be circumstellar disks, discussing how they give constraints on the available models of disk formation and evolution, and then giving detailed description of the scenario he thought best, including its predictions and possible observational tests. He would have covered topics over a wide area and connected them in an unexpected way to understand the physics of circumstellar disks around active OB stars.

In this talk, I tried to keep the structure I imagined John would have had. However, because of the limitation of my knowledge, I have to admit that my talk is inevitably less comprehensive, covering narrower topics, not to mention the lack of his wits, and maybe a little bit biassed to the theoretical model called the viscous decretion disk model, which I have been working on for the last several years. For a more comprehensive and orthodox review, I refer the readers to his recent article with Thomas Rivinius on classical Be stars (Porter & Rivinius 2003).

There are several classes of early type stars with optical emission lines originated from circumstellar disks. They include Herbig Ae/Be stars, Algol systems, σ Ori E and similar objects, supergiant B[e] stars, and Be stars. These stars have different physics of disk formation. Circumstellar disks in Herbig Ae/Be stars and Algol systems are accretion disks. The disk around σ Ori E is a stellar wind guided and confined by a strong magnetic field (Townsend et al. 2005; ud-Doula et al. 2006). The origin of the circumstellar disk around supergiant B[e] stars is not well understood. Both an equatorial wind produced by wind bi-stability (Lamers & Pauldrach 1991) and a Keplerian viscous disk (Lee et al. 1991) have trouble in accounting for observations (Porter 2003). As discussed in detail in next sections, the circumstellar disk around Be stars is likely produced by the effect of viscosity. In this talk, I put the focus only on the physics of the circumstellar disk around Be stars.

A classical definition of Be stars is non-supergiant early-type stars, which show, or at some time showed, the Hα line in emission. The spectral type of Be stars ranges from late O- to early A-type. As discussed in more detail by Porter & Rivinius (2003), this definition, which is still used today, is rather unsatisfactory, because it does not exclude systems with accretion disks/streams such as Herbig Ae/Be stars and Algol systems and magnetically-confined disk systems such as σ Ori E. Below I consider only "classical" Be stars.

A Be star has a two-component extended atmosphere, a polar region and a cool ($\sim 10^4\,\mathrm{K}$) equatorial disk. The former consists of a low-density, fast ($\sim 10^3\,\mathrm{km\,s^{-1}}$) outflow emitting UV radiation. The wind structure is well explained by the line-driven wind model (Castor et al. 1975; Friend & Abbott 1986; see also Krtička & Kubát 2007). On the other hand, the equatorial disk consists of a high-density plasma, from which the optical emission lines and the IR excess arise. For many years, the formation of Be star disks were one of the most challenging problems in the study of Be stars. So, it is not surprising that most theoretical work done in 1970's through 1980's were kinematical. For the last 15 years, however, several *dynamical* disk models have been proposed. Now the model called the viscous decretion disk model seems to have survived and researchers' preference is converging to this model. In this review, I first discuss the key observational features of Be circumstellar disks. Then I explore the viscous decretion model and its predictions in detail, based on numerical simulations.

2. Key Observational Features of Be Circumstellar Disks

Until quite recently, theoreticians and observers have worked on the disk business rather separately. This is in contrast to the way of research in neighboring areas such as the stellar wind business and the non-radial pulsations business, where fruitful collaboration between theoreticians and observers has often been seen. The lack of fruitful collaboration in the disk business was largely due to the lack of a reliable disk model, on which further progress should be developed. During this unfortunate period, the driving force that has developed our understanding of the Be circumstellar disks was observations. In this section, we discuss several key features obtained by observations, which put important constraints on the structure and dynamics/kinematics of the Be circumstellar disks.

2.1. Geometry

Although the Be circumstellar gas had been suspected to be disk-like for many years, it was relatively recent that it was confirmed by observations. The first indication that the envelope is not spherically symmetric came from polarization observations (e.g., McLean & Brown 1978). A more direct evidence was obtained by observations with optical interferometers (e.g., Quirrenbach et al. 1994). The interferometric images of some Be stars agreed with the disk-like structure with inclination angles estimated from the spectral shape of the emission lines.

In order to study how thin/thick the Be disk is, the opening angle of the disk has often been used. For example, from statistical study of shell stars (Be stars seen nearly edge-on), Porter (1996) and Hanuschik (1996) obtained that the opening angle is 5° and 13°, respectively. These studies showed that the Be disks are geometrically thin. To be frank, however, the opening angle is a rather poor tool to probe the detailed disk structure, because it depends on the local physical quantities as well as the distance from the star.

2.2. Basic Kinematics

From spectroscopic observations alone, it was not easy to determine whether the rotation law of the Be circumstellar disk was Keplerian, angular momentum conserving or of a type in between. The breakthrough came with the optical interferometric observations. The relationship between the disk size resolved with the optical interferometers (Quirrenbach et al. 1994, 1997) and the separation of the double peaks of the peaks of the Hα line was in agreement with that expected for a Keplerian disk. Analyzing the spectral line profiles, Hummel & Vranken (2000) that the rotation velocity of the Be disk varies as r^{-j} with $j < 0.65$, where r is the distance from the star. With other observational (and theoretical) features discussed below, the Be disk is now considered to be nearly Keplerian.

The specific angular momentum in the Keplerian disk increases as $r^{1/2}$, and the Be disk is formed from inside by material ejected from the central star. This means that the angular momentum has to be somehow transferred from the inner part to the outer part. Given that the angular momentum transfer is likely to take much longer than the sound crossing time, it is also likely that the outflow in the Be disk is highly subsonic. This theoretical expectation has been confirmed by observations. Hanuschik (1994, 2000) and Waters & Marlborough (1994) showed that the radial velocity of the disk is smaller than a few $\mathrm{km\,s^{-1}}$, at least within ~ 10 stellar radii. This upper limit provides another important constraint on the disk model.

2.3. Long-Term V/R Variations: The Most Stringent Constraint on the Disk Kinematics

Many Be stars exhibit long-term, quasi-cyclic variations in the relative intensities of the violet and red components in double-peaked, Balmer emission-line profiles. This phenomenon is called the long-term V/R variation. The periods of V/R variations range from years to a decade, which is more than 10^2 times as long as the rotation period at the outer radius of the Hα emitting region.

The long-term V/R variations were once a long-standing enigma. Today, they are attributed to global disk oscillations with a one-armed, density perturbation pattern. This possibility was first pointed out by Kato (1983) and later applied to Be disks by Okazaki (1991). Then, Papaloizou et al. (1992) proposed that the quadrupole contribution to the potential of the rapidly-rotating, deformed star confines the oscillation mode to the inner part of the disk. They also pointed out that the confined mode should precess in the prograde direction. Since then, observational evidences supporting the global disk oscillation model (or one-armed oscillation model) have been accumulated (e.g., spectroscopic study by Hummel & Hanuschik 2000; interferometric study by Vakili et al. 1998). The model qualitatively explains the basic features of long-term V/R variations. It should be noted, however, that the mechanism(s) to confine the one-armed mode is still an open issue (e.g., (Okazaki 1997)). The optically thick line forces could work in the Keplerian shear flow to confine the mode (Gayley et al. 2001). This uncertainty about the confining mechanism(s) makes the quantitative comparison between the model and observations difficult and less reliable.

In the one-armed oscillation model, the slow pattern speed of the one-armed (i.e., $m = 1$) density wave results from a slight ($< 1\%$) deviation of the rotation velocity distribution in the disk from the Keplerian distribution. As a result, the model imposes the most stringent constraint on the rotation velocity distribution of any disk model. On the other hand, owing to the extreme sensitivity on disk rotation, a tiny change of a disk parameter and/or a stellar radiation effect can easily change the oscillation period significantly. This is another reason why it is difficult to construct a quantitatively reliable model.

The one-armed oscillation model imposes another constraint on the disk kinematics. As shown by Okazaki (2000), the mode confinement occurs only in the region where the radial flow is highly subsonic ($\lesssim 1\%$ ($\lesssim 1\%$ of the sound speed). Thus the presence of the one-armed oscillation mode is the evidence that the Be disk is a nearly both the deviation from the Keplerian rotation and the ratio of the radial velocity to the sound speed are $\lesssim 1\%$. Any satisfactory disk model has to satisfy this condition.

2.4. Other Global Disk Variations

The circumstellar disk of a Be star, if it is once persistent, can be lost completely and reformed, thus changing the star's appearance from a Be star to a normal B star and to a Be star again (e.g., θ θ CR, o And). In some other cases, the disk loss occurs partially and then the disk grows again. Such a change occurs on a time-scale of years to decades. At present, no available model can fully explain this large scale variation of the disk state. The mechanism responsible for the disk loss could be re-accretion by viscosity, radiation-driven ablation, or the entrainment by the stellar wind, although the time-scales of disk formation and/or loss seem consistent with the viscous time-scale.

Occasionally, the Be disk exists only transiently [e.g., μ Cen in 1977–1989 (e.g., Rivinius et al. 1998)]. Kroll & Hanuschik (1997) studied the evolution of the gas explosively ejected from a Be star to model the transient disk formation, using a three-dimensional, Smoothed Particle Hydrodynamics (SPH) code. They found that the gaseous particles gradually expand and form a transient, nearly Keplerian disk in the viscous time-scale.

Some Be stars show the loss and reformation of the high-velocity wings of emission lines, which indicates the loss and reformation of the inner part of the disk [e.g., η Cen (Rivinius et al. 2001)], or more dramatically double pairs of double-peaked emission lines, an indication of the presence of two rings/disks [e.g., X Per (Tarasov & Roche 1995; Clark et al. 2001)]. Although the scale of the variation is small compared to the complete disk loss and reformation, the basic mechanism for both phenomena could be the same.

2.5. Density and Temperature Distribution

Until quite recently, the density and temperature distributions in the Be disk have been studied separately. A constraint on the density distribution has been obtained, using the IR excesses arising from the Be disk. Assuming that the disk has a power-law density distribution $\rho \propto r^{-n}$ with a constant opening angle, Waters (1986) made model fits to the IR continuum data from four Be stars and found that the index n is in the range $2 < n < 4$. Note that this range of density distribution indicates the accelerating radial velocity in the disk. Subsequent studies also found the density index in the similar range. Adopting a nearly Keplerian disk with hydrostatic balance in the vertical direction, Porter (1999) also found the similar range of density distribution. Given that the viscous decretion disk model discussed in the next section predicts $n = 7/2$ if the disk is isothermal, the observed flatter density distribution suggests the decrease in disk temperature with radius (Porter 1999). Note also that any satisfactory disk model has to explain why Be stars have this range of density distribution.

When modeled, the Be disk has often been assumed to be isothermal at $0.5 - 0.8T_{\rm eff}$ with $T_{\rm eff}$ being the stellar effective temperature. Balancing the local energy gain and loss due to radiative processes, Millar & Marlborough (1998, 1999) found that the isothermal assumption is a good first approximation except in a region close to the star, where the disk midplane temperature is significantly lower than the temperature of the disk upper atmosphere. Recently, using three dimensional NLTE Monte Carlo simulations, Carciofi et al. (2007) obtained the self-consistent, density and temperature structure of the viscous decretion disk around Be stars. They confirmed that the disk is approximately isothermal in the optically-thin region. They showed, however, that the disk midplane temperature has a steep radial gradient in the optically thick region near the star and an isothermal disk model is far from correct in this region.

3. Viscous Decretion Disk Model

For the last fifteen years, several disk formation models have been proposed. According to those models, the Be disk is formed by viscous diffusion (Lee et al. 1991), by wind compression (Bjorkman & Cassinelli 1993), or by magnetic channeling (Cassinelli et al. 2002). Among them, only the viscous disk model can make a nearly Keplerian disk with highly subsonic radial flow.

3.1. Basic Scenario

The viscous decretion disk model assumes that the star can eject material with the Keplerian velocity at the stellar equatorial surface. The ejected material then drifts outwards because of the viscous effect and forms a disk. The basic equations for viscous decretion disks are the same as those for viscous accretion disks, except that the sign of $\dot{M}$ (mass decretion/accretion rate) is opposite. The boundary conditions for decretion disks, however, are different from those for accretion disks. Therefore, the decretion disk has a structure different from that of the accretion disk (Pringle 1991). It is also important to note that the decretion disk is never steady. It either grows, when the star exerts a positive torque on it, or decays, when no/negative torque is exerted, although there are mathematically steady disk solutions with infinite mass or divergent radial velocity.

3.2. One-Dimensional Simulation of Viscous Disk Evolution around Isolated Be Stars

The easiest way to see global characteristics of the structure and evolution of a viscous decretion disk is to perform one-dimensional simulations. For this purpose, we assumed that the disk around an isolated Be star is axisymmetric and Keplerian and in hydrostatic equilibrium in the vertical direction. We took a B0V star with $M_* = 18M_\odot$, $R_* = 8R_\odot$ and $T_{\rm eff} = 26000\,{\rm K}$ as the Be star, and assumed the Be disk to be isothermal at the temperature of ${}^1/_2 T_{\rm eff}$. We adopted the Shakura-Sunyaev's viscosity parameter $\alpha_{\rm SS} = 0.1$. The evolution of such a disk is described by

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[\frac{\dfrac{\partial}{\partial r}\left(\alpha_{\rm SS} c_{\rm s}^2 r^2 \Sigma\right)}{\dfrac{d}{dr}\left(r^2\Omega\right)}\right] \tag{1}$$

with

$$V_{\rm r} = -\frac{\dfrac{\partial}{\partial r}\left(\alpha_{\rm SS} c_{\rm s}^2 r^2 \Sigma\right)}{r\Sigma\dfrac{d}{dr}\left(r^2\Omega\right)} \tag{2}$$

(e.g., Pringle 1981), where Σ is the surface density, $V_{\rm r}$ is the radial velocity, $c_{\rm s}$ is the isothermal sound speed, and $\Omega = (GM_*/r^3)^{1/2}$ is the angular frequency of disk rotation.

In order to create a situation similar to that in the SPH simulations shown below, we injected mass at a constant rate at $r_{\rm inj} = 1.02R_*$. At the inner boundary $r = R_*$, we adopted the torque-free boundary condition, $\Sigma = 0$. We also took $\Sigma = 0$ as the outer boundary condition at $r = 10^3 R_*$, which affected the disk structure only in a region near $r = 10^3 R_*$.

The evolution of the surface density Σ for the initial ten years is shown in the left panel of Fig. 1, while that of the radial velocity V_r normalized by the isothermal sound speed in the right panel. The time interval between adjacent contours is 1 yr. It should be noted that the density distribution becomes flatter and the radial velocity, which increases with radius, decreases as the disk grows. It should also be noted that, as mentioned in the previous section, no steady solution exists for decretion disks unlike for accretion disks (Pringle 1991). Instead, the formal solution of equation (1) with $\partial\Sigma/\partial t = 0$ and $V_{\rm r} = 0$ gives the disk structure at $t \to \infty$, which is given by $\Sigma \sim r^{-2}$ in our isothermal disk model.

3.3. Three-Dimensional SPH Simulation of Viscous Disk Evolution around Isolated Be Stars

Simulations presented below were performed with a three-dimensional SPH code. The artificial viscosity parameters varied with time and space so as to emulate the Shakura-Sunyaev's viscosity prescription with $\alpha_{\rm SS} = 0.1$. To model the mass ejection from the Be star, we injected isothermal particles rotating at the

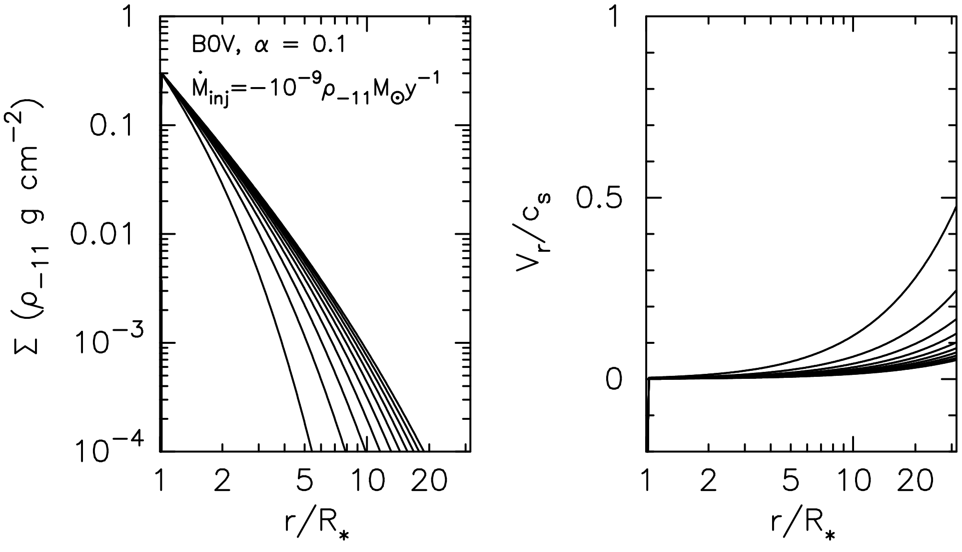

Figure 1. The initial ten-year evolution of the one-dimensional, viscous decretion disk: (left) the surface density and (right) the radial velocity. The surface density is normalized so that the highest local density at $t = 1\,\mathrm{yr}$ becomes $10^{-11}\mathrm{g\,cm^{-3}}$, a typical value for Be stars (Waters et al. 1988). The time interval between adjacent contours is 1 yr. Time runs from left to right in the left panel and from top to bottom in the right panel. $\dot{M}_{\rm inj}$ is the mass injection rate from the central star.

Keplerian angular velocity at the stellar equatorial region into a radius just outside the stellar surface. We took the same parameters for the Be star and Be disk as in the above one-dimensional simulation. For more details of the model, see Okazaki et al. (2002).

Fig. 2 shows the surface density evolution for the first three years (left) and the disk structure at $t \sim 3.5\,\mathrm{yr}$ (right). The time interval between adjacent contours in the left panel is half a year. In the right panel, the solid, the dashed, and the dash-dotted lines denote the surface density, the radial Mach number, and the azimuthal velocity normalized by the stellar critical velocity, respectively. On the figure, the profile of V_ϕ is indistinguishable from the Keplerian rotation distribution. In this simulation with an axisymmetric mass input, the disk evolution is similar to that in the one-dimensional, diffusion-type simulation.

From Fig. 2, we observe that the disk structure is nearly Keplerian and the radial velocity, which decreases with time, is highly subsonic for $r \lesssim 10R_*$ at $t \gtrsim 1\,\mathrm{yr}$. These features are in agreement with the observed key features of Be stars. On the other hand, the surface density distribution is very steep soon after the beginning of disk formation. With time it becomes flatter toward the asymptotic distribution $\Sigma \propto r^{-2}$. This means that the density index n in isothermal disks is always larger than 7/2, which is much steeper than the observed range $2 < n < 4$ (see section 2.5.). This inconsistency between the isothermal disk model and the IR excess observations suggests that the assumption of isothermality is wrong and the temperature steeply decreases with radius in the inner part of the disk, as shown in recent, 3D NLTE Monte Carlo simulations (Carciofi et al. 2007).

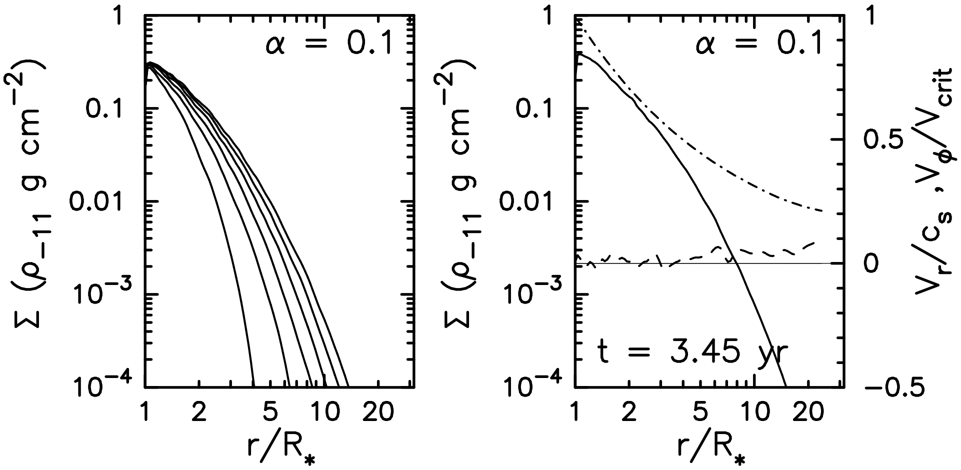

Figure 2. Evolution of the viscous decretion disk around an isolated Be star in a 3D SPH simulation with $\alpha_{\rm SS} = 0.1$. In the left panel, the surface density evolution for the first three years is shown by the solid lines, the interval of which is half a year. The surface density is measured in units of $\rho_{-11}\,{\rm g\,cm^{-2}}$, where ρ_{-11} is the highest local density at $t = 1\,{\rm yr}$ normalized by $10^{-11}\,{\rm g\,cm^{-3}}$. The right panel shows the disk structure at $t \sim 3.5\,{\rm yr}$. The solid, the dashed, and the dash-dotted lines denote the surface density, the radial Mach number, and the azimuthal velocity normalized by the critical velocity of the Be star. In both panels, the density is integrated vertically and averaged azimuthally, while the velocity components are averaged vertically and azimuthally. The number of SPH particles at the end of the simulation was about 2×10^4.

Apart from this numerical discrepancy, the basic trend in the change of density distribution is likely universal in any viscous disk model. Thus it is important to analyze the IR excess data to see whether or not the density index decreases with time. If the observed density index for *young* Be disks does not decrease with time, then the viscous decretion disk model will be safely ruled out.

3.4. Effect of the Stellar Torque

As mentioned in section 2.4., some Be stars show the loss and reformation of the inner part of the disk. It is interesting to see whether the viscous decretion disk model can explain such a phenomenon. In order to emulate the effect of the intermittent mass supply from the star, we continued the SPH simulation shown in Fig. 2, varying the mass injection rate into the inner disk radius. In this simulation, the mass injection rate was kept constant for $t \lesssim 3.5\,{\rm yr}$ (Fig. 2), then set to zero for $3.5 \lesssim t \lesssim 6.9\,{\rm yr}$ (left panel of Fig. 3), and resumed at the same level as for the first 3.5 yr (right panel of Fig. 3).

The left panel of Fig. 3 shows that, once the mass supply from the star is shut off, the accretion begins at the innermost region and the accreting region gradually propagates outward. Since the viscous time-scale is proportional to $r^{1/2}$ in isothermal disks, the gas in the outer part of the disk does not notice the turning-off of the mass supply and keeps diffusing outward by viscosity until

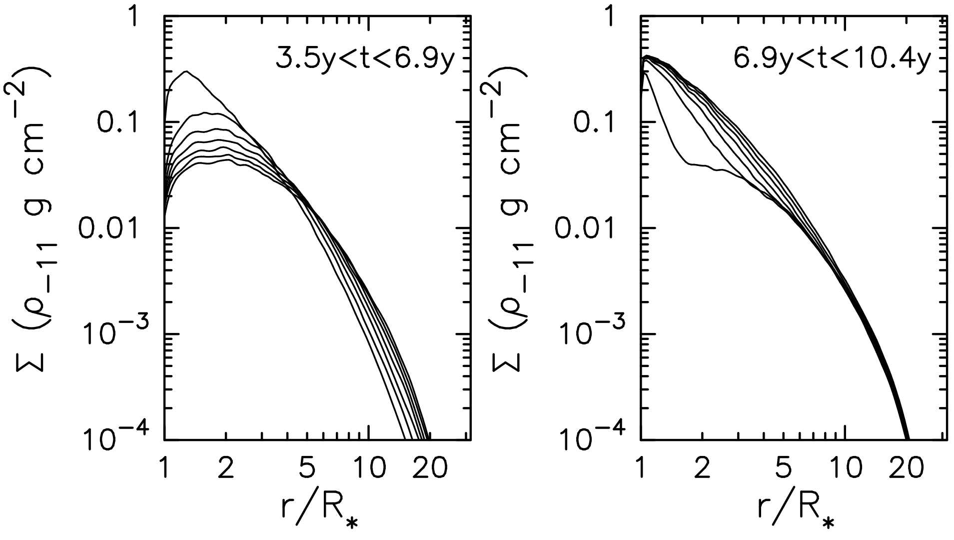

Figure 3. Surface density evolution of the viscous disk around an isolated Be star for $\alpha_{\rm SS} = 0.1$. After a constant mass supply from the star for $t \lesssim 3.5\,{\rm yr}$ (Fig. 2, it was shut off for $3.5\,{\rm yr} \lesssim t \lesssim 6.9\,{\rm yr}$ (left panel), and then resumed at the initial rate (right panel). The format of the figure is the same as that of the left panel of Fig. 2. Time runs from top to bottom in the left panel and from bottom to top in the right panel.

$t \sim r^{1/2}$. Such a disk will be observed as a broad ring (or a disk with an inner cavity) with the peak density decreasing with time.

When the mass supply from the star is resumed, the reformation of the inner disk begins. For a period less than a year (for $\alpha_{\rm SS} = 0.1$), the inner disk has a much steeper density distribution than the outer disk does. Such a density distribution could temporarily exhibit two sets of double-peaked line profiles such as those observed in X Per (Tarasov & Roche 1995; Clark et al. 2001).

4. Concluding Remarks

Until recently, the understanding of the Be circumstellar gas has been mostly through observations. Based on the observational constraints on geometry, kinematics and structure in density and temperature, it has been established that the Be circumstellar disk is a geometrically-thin, nearly-Keplerian disk with a highly subsonic radial flow. Recently however, a reliable theoretical model called the viscous decretion disk model has emerged. Even in a naive form currently available, this model can qualitatively explain all the key disk features. Thus the viscous decretion disk model has enabled us to study both theoretically and observationally the structure and dynamics of the Be disk more deeply. Computer codes for the radiative transfer simulations as well as hydrodynamic/magnetohydrodynamic simulations are now available. Therefore, it is ripe for observers and theoreticians to work together to construct the whole disk story.

Although the viscous decretion disk model is found to be most satisfactory, it simply assumes a super-Keplerian rotation of the equatorial region of the star to lift up mass and, therefore, still lacks the physics of mass and angular momentum input to the inner disk. Understanding the mass ejection mechanism from the star is vital for the model to be really reliable.

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Discussion

W. Hummel: Would it make sense to use a non-axisymmetric mass-input rate for the decretion disk simulations, and could non-axisymmetric mass input at the inner radius induce a global oscillation mode?

A. Okazaki: The disk evolution would not be affected by the mode of the mass-input rate, as long as the time-scale of the change in the mass-input is much shorter than the disk evolution time-scale. The non-axisymmetric mass input, however, could affect the global oscillation mode, giving a large initial amplitude to the $m = 1$ perturbation component. It could also make the $m = 1$ mode grow faster, given that the growth time-scale of the $m = 1$ mode is the viscous time-scale and the interaction of the non-axisymmetric mass input with the inner part of the disk would enhance the effective viscosity.

S. Štefl: In your paper published in 1997, you compared the effects of rotational distortion of the central star and weak lines and concluded that the rotational distortion can confine the $m = 1$ oscillations in disks of late Be stars, but plays negligible role in early Be stars. However, you used an approximation valid only for small quadrupole distortions of the gravitational potential. Can you comment on how your conclusions may change by the inclusion of near-critical rotation, such as what is indicated by α Eri interferometry and theoretical work by Townsend et al. (2004)?

A. Okazaki: If the rotation of the star is nearly critical and if the apsidal motion constant is as large as 10^{-2}, I think the effect by the rotational distortion of the star can confine the $m = 1$ mode to the inner part of the disk even in early Be stars.

Ph. Stee: You said it should be nice to have some predictions and constraints from observations, but you did not mention the interferometric observations of

γ Cas by Bério et al. (1999) nicely compatible with a one-armed oscillation with a 7-year period.

A. Okazaki: I should have mentioned that, but what I want to emphasize here is that the one-armed oscillation model is quantitatively not reliable yet. This is mainly because the model still lacks the reliable mode-confining mechanism(s).

S. Owocki: In dense disks, it is unlikely that an optically thin line-force could be a proper description of the effect of radiation forces in perturbing the $1/r$ potential to give the one-armed mode precession. Thus Ken Gayley and I examined how optically thick forces could operate, accounting for the desaturation of the non-radial rays due to the Keplerian shear. For an analysis of any slightly elliptical orbits, we found two effects, one leading to prograde precession, and the other to retrograde. Unfortunately, the one that seemed to dominate was the wrong one, compared to observations. It's possible this could change with large ellipticity, but I think a most likely case is non-$1/r$ forms in gravity due to oblateness of a near-critically rotating star.

H. Henrichs: In X Per, the profiles in lines other than Hα show different structures. Could you comment?

A. Okazaki: If the period of the variation of those lines is the same as that of Hα, it could be caused by an overtone of the mode.

J. Fabregat: In X Per, the Hβ and He lines have a smaller equivalent width than the Hα emission line. This makes the structure of the lines look somewhat different. The main pattern of V/R variation, however, is the same in all lines.

A. Kaufer: Do we also have some observational evidence for material falling back once the outburst is stopped?

D. Baade: His simulation of a temporary shut-off of the mass loss is qualitatively in good agreement with what Thomas Rivinius, Stan Štefl and I have observed in some Be stars. The inner disk is cleared, the outer one changes but little. If the time before consecutive outbursts is large enough ($\sim$ a year), there is probably an initial density minimum between the new inner disk and the old outer one.

K. Bjorkman: For the polarization outbursts, we see a rise and then decline of the polarization level on a time-scale of about 4–6 weeks. This presumably reflect what is happening in the inner part of the disk